DATA

I use this randomly generated super symmetric solution for the analysis:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

We call this sudoku M

ANALYSIS

There exists exactly 4 initial choices for the first set of comparison numbers when checking for a symmetry [I J K L]. Given M, the 4 possible first comparison numbers are:

123,147,168,159

All symmetries of the form \*\*\*1 make their first set of comparisons with the highlighted numbers:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

All symmetries of the form \*\*\*2 make their first set of comparisons with the highlighted numbers:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

All symmetries of the form \*\*\*3 make their first set of comparisons with the highlighted numbers:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

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All symmetries of the form \*\*\*4 make their first set of comparisons with the highlighted numbers:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

We have effectively partitioned the set of all possible starting comparisons by noting that the comparison set is completely determined by the last number.

Proof of the following statement:

By definition, all 4 numbers/letters that define the symmetry must be unique and either 1, 2, 3, or 4. All symmetries thus contain all 4 integers. If integer x is the last integer, it may not be the first, second, or third integer in the symmetry. We have shown that all \*\*\*x symmetries make comparisons starting with exactly one set of three numbers. This implies that if 1, 2, 3, or 4 are present in any spot of the symmetry, excluding the last spot, the first comparison made will not utilize the set defined by \*\*\*x where x is all 1, 2, 3, 4 not in the last spot.

I claim that the location of comparison is determined solely by the first number.

If my claim holds, than the first and last numbers together determine the direction of comparison as well as the sets of comparisons.

1234:

We know that \*\*\*4 implies that 1234 is being compared, but to what? Let green be the first comparison set, blue be the second, and violet be the third.

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

1324:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

1243:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

1423:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

1342:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

1432:

1 2 3 6 4 5 8 9 7

4 5 6 9 7 8 3 1 2

7 8 9 2 3 1 5 6 4

5 6 4 7 8 9 2 3 1

8 9 7 1 2 3 6 4 5

3 1 2 4 5 6 9 7 8

9 7 8 3 1 2 4 5 6

2 3 1 5 6 4 7 8 9

6 4 5 8 9 7 1 2 3

Let us note that if 1 ab x holds, so too will 1 ba x.

To prove this, it is easy to show that for each choice of ab, ba makes the same essential comparisons, albeit in a different order.

As a result of the above, we may claim that 1 \*\* x will compare the set of numbers defined by x to other sets within 1.

I believe it to be the case that w \*\* x implies the set of numbers defined by x will be compared to sets of numbers within w.